A Benchmark Problem on Structural Health Monitoring of High-Rise Slender Structures

Phase I: Field vibration measurement and model updating

Description of the FE model and model reduction

The Full Finite Element (FE) Model

Based on design drawings of the TV tower, a fine 3D finite element (FE) model has been established in ANSYS, as shown in Fig. 1. The 3D full-order model contains 122,476 elements, 84,370 nodes, and 505,164 degrees-of-freedom (DOFs) in total. In the model, PIPE16 and BEAM44 (2-node 3D beam elements with six DOFs at each node) are employed to model the outer structure, antenna mast, and connection girders between the inner and outer structures. Four-node and three-node shell elements with six DOFs at each node are used to model the shear walls of the inner structure and floor decks.

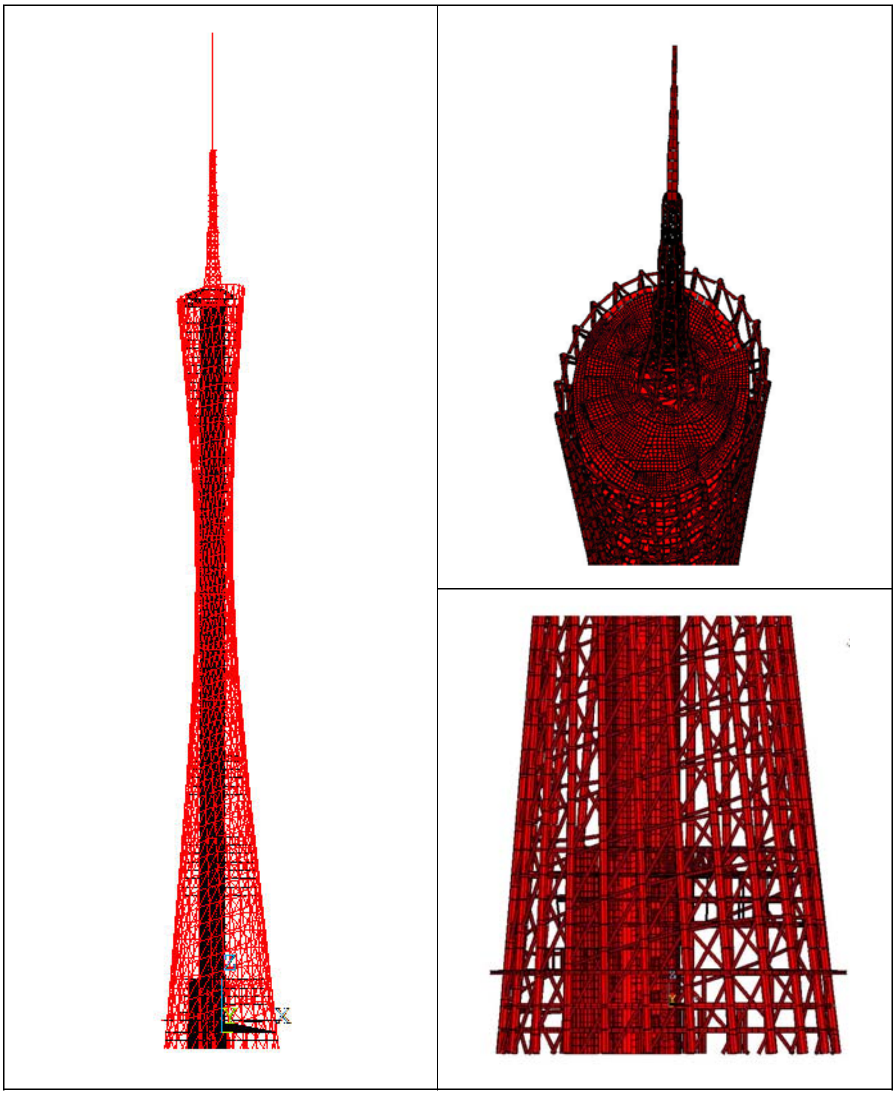


Fig. 1 3D model of Guangzhou New TV Tower in ANSYS

Reduced FE Model

The size of the 3D full model is too large for SHM and related studies. A reduced 3D beam model is established based on the full model.

The entire structure is divided into 37 segments, each of which is modelled as a linear elastic beam element. Consequently the structure is modelled as a cantilever beam with 37 beam elements and 38 nodes. The main tower consists of 27 elements and the mast is modelled as 10 elements. As shown in Fig. 2, the nodal number increases from 1 at the fixed base to 38 at the free top end. The vertical coordinates of the nodes are listed in Table 1. As the geometric centroid of each floor varies, the beam of the reduced model aligns with the centriodal axis of the tower mast. The vertical displacement is disregarded in the reduced model and thus each node has 5 DOFs, i.e., two horizontal translational DOFs and three rotational DOFs. As a result, each element has 10 DOFs and the entire model has 185 DOFs in total. The coordinate system of the reduced model is illustrated in Fig. 2.

Table 1 The nodal coordinate (z) of the reduced model

Node number	z (m)	Node number	z (m)
1	-10.00	20	355.05
2	0.00	21	375.85
3	12.00	22	381.20
4	22.25	23	396.65
5	27.60	24	407.05
6	58.65	25	417.45
7	84.65	26	427.85
8	95.05	27	438.25
9	105.45	28	443.60
10	116.20	29	480.00
11	147.05	30	497.60
12	157.45	31	505.20
13	168.00	32	520.70
14	204.25	33	531.20
15	225.20	34	545.20
16	272.00	35	565.20
17	308.25	36	580.70
18	329.20	37	598.00
19	344.65	38	618.00

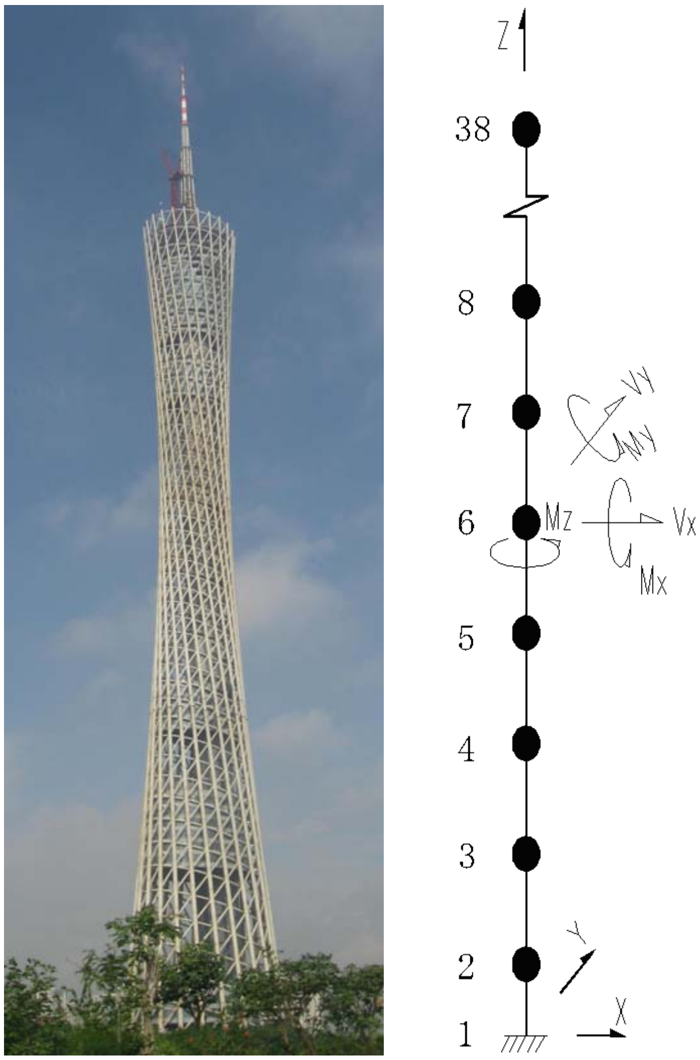


Fig. 2   The TV Tower and reduced FE model

Half mass of each segment between two adjacent nodes (including floors, if any) is added to the upper node and the remaining half added to the lower node. The equivalent rotational inertia of the segment with respect to the node is also calculated, and half is distributed to the upper node and half to the lower node. The element stiffness matrix $\mathbf{K}^e$ of the reduced model is formulated by the displacement method through the following procedures:

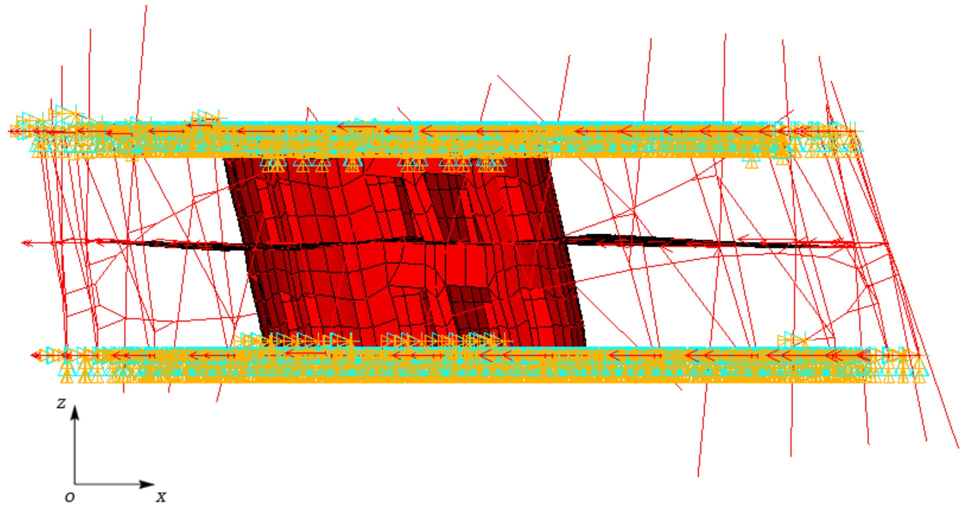
(i) For each beam element, the corresponding segment between two horizontal sections is selected from the full-order model. All of the nodes on the two sections are constrained.

(ii) To calculate the stiffness coefficients of the *j*th column of $\mathbf{K}^e$, a positive unit displacement is imposed on the *j*th DOF of the segment while all other possible displacements are prevented. Here j = 1, 2, …, 10. j = 1 to 5 refer to the lateral translational displacement in x and y direction, and rotation along x, y and z axes at the lower node, respectively. Similarly j = 6 to 10 refer to the displacements at the upper node. Fig. 3(a) illustrates the case of j = 1, where all nodes at the upper section are constrained and all nodes at the lower section are imposed on by a unit displacement in x direction simultaneously. The cases of j = 2 to 5 are shown in Figs. 3(b) to 3(e), respectively. It is noted that when j = 3, 4, and 5, the corresponding translational displacement should be first calculated and applied to the nodes.

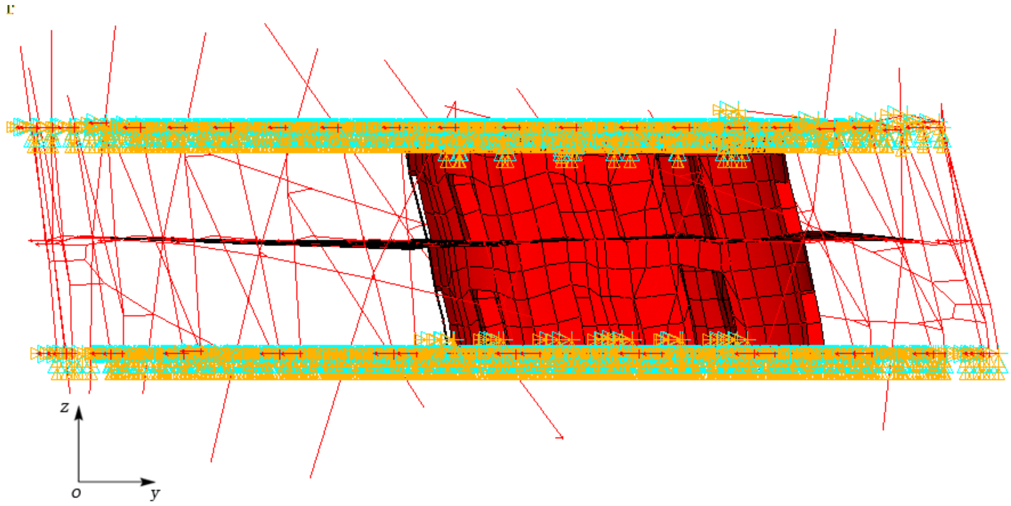
(iii) The resultant generalized force at the *i*th DOF of the segment due to the unit displacement at the *j*th DOF is calculated and denoted as $\mathbf{K}_{ij}$ of the beam element. Here i = 1, 2, …, 10. Correspondingly, i = 1 to 5 refer to the shear forces at x and y direction, and moments (torque) along x, y and z axes at the lower node, respectively. i = 6 to 10 refer to the force components at the upper node.

(iv) The above procedures (i) to (iii) are repeated for each element and all the element stiffness matrices are obtained accordingly.

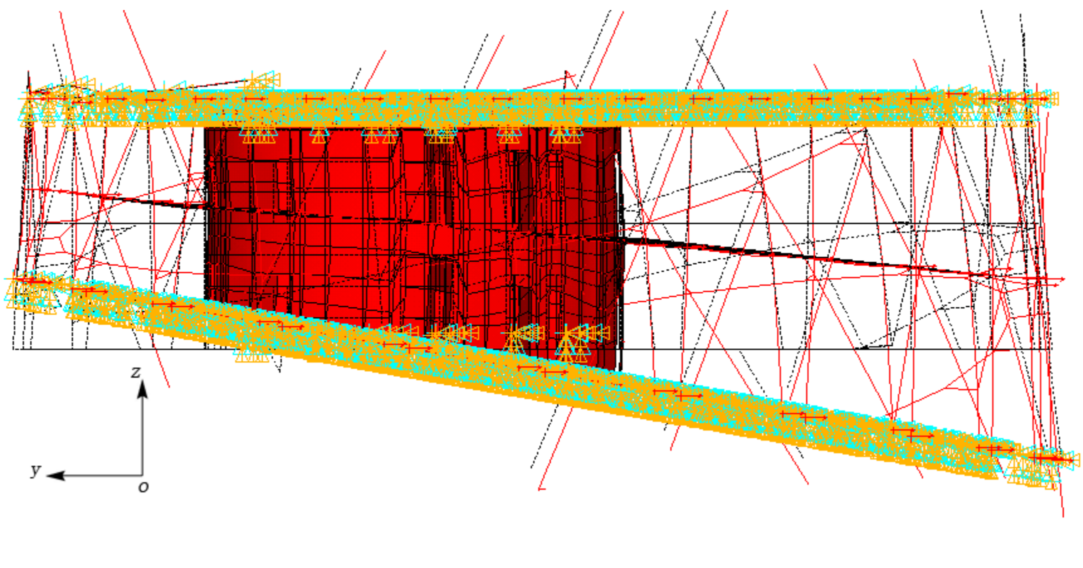
(v) The element stiffness matrices are assembled to obtain the global stiffness matrix of the reduced model.



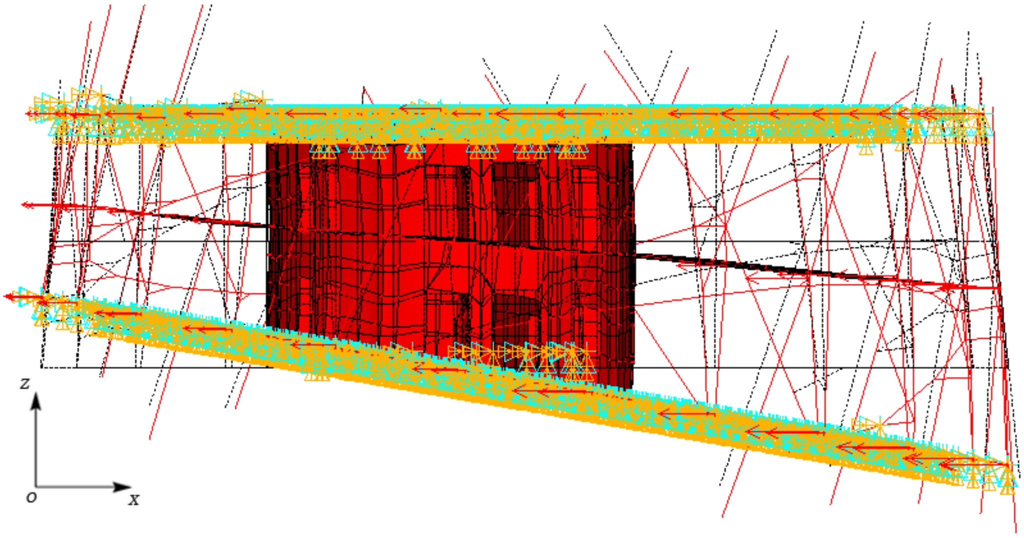
(a)



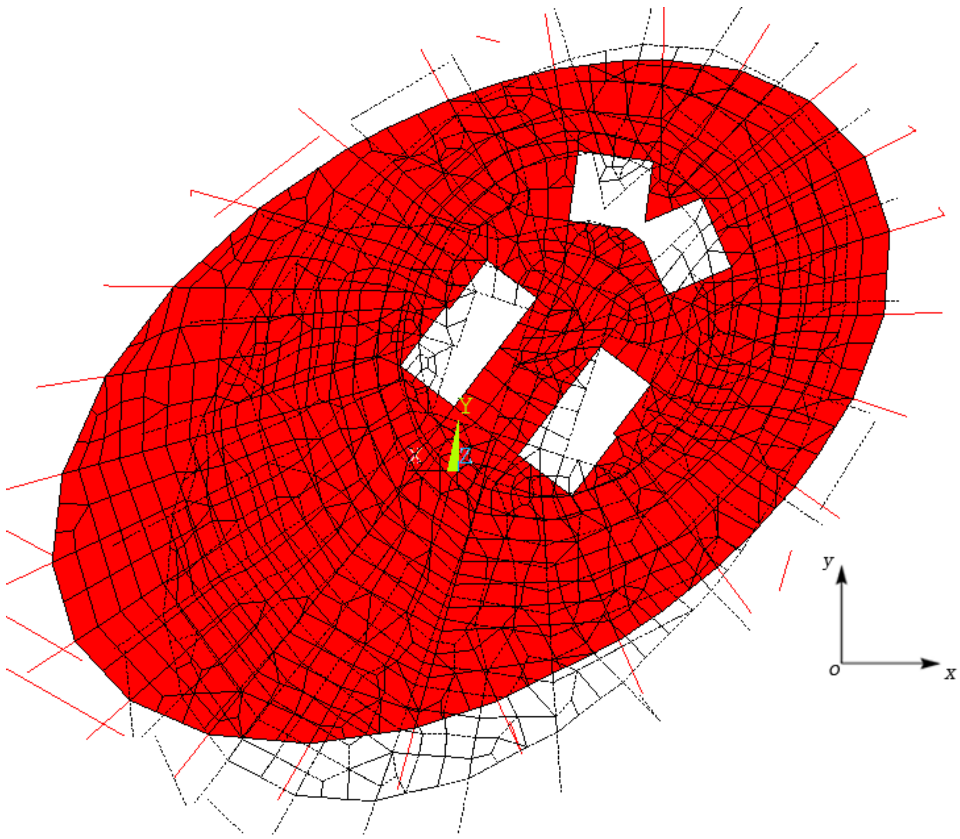
(b)



(c)



(d)



(e)

Fig. 3 Unit positive displacement in different directions:
(a) x direction (j = 1); (b) y direction (j = 2); (c) rotation along x axis (j = 3); (d) rotation along y axis (j = 4); (e) rotation along z axis (j = 5)

Model Tuning

The dynamic properties of the reduced model inevitably differ from those of the full model due to simplification. The reduced model is fine tuned so that its dynamic characteristics match those of the full model as closely as possible. The following residual vector is to be minimized:

$$\mathbf{R} = \{\mathbf{R}^{\mathrm{Q}} \qquad \mathbf{R}^{\mathrm{S}}\}^{T} \tag{1}$$

where $\mathbf{R}^{\mathrm{Q}}$ and $\mathbf{R}^{\mathrm{S}}$ represent the frequency differences and mode shape similarities between the full-order model and reduced-order model, which are respectively expressed as

$$\mathbf{R}^{\mathbf{Q}}(i) = \frac{\left| f_i^{\mathrm{F}} - f_i^{\mathrm{R}} \right|}{f_i^{\mathrm{F}}} \tag{2}$$

$$\mathbf{R}^{\mathbf{S}}(i) = 1 - MAC(i) \tag{3}$$

$$MAC(i) = \frac{\{\varphi_i^{\mathrm{F}}\}^T \{\varphi_i^{\mathrm{R}}\}}{\left(\{\varphi_i^{\mathrm{F}}\}^T \{\varphi_i^{\mathrm{F}}\}\right)\left(\{\varphi_i^{\mathrm{R}}\}^T \{\varphi_i^{\mathrm{R}}\}\right)} \tag{4}$$

where f_i denotes the ith frequency, φ_i is the ith mode shape, the superscripts ‘F’ and ‘R’ represent the items associated with the full model and the reduced model, respectively, and ‘T’ denotes vector (matrix) transpose.

Different weighting factors are assigned to different residual items according to their importance to the dynamic properties of the structure. The objective function J is defined by the following equation

$$J = \mathbf{R}^T \mathbf{W} \mathbf{R} \tag{5}$$

where $\mathbf{W}$ is a diagonal weighting matrix. The modal frequencies and mode shapes of the first fifteen modes are included in the updating. The weight coefficients are set to 10 for all frequencies ($\mathbf{R}^{\mathbf{Q}}$), 1.0 for the first four bending mode shapes and the first two rotational mode shapes ($\mathbf{R}^{\mathrm{S}}$), 0.5 for the next four bending mode shapes, and 0.3 for other five higher bending mode shapes. As the full model contains many nodes at each floor and their mode shape components may differ with each other, the mode shape values are averaged and treated as the mode shape component of the corresponding node in the reduced model.

As mentioned above, the Guangzhou New TV Tower is unsymmetric and the geometric centroid of each floor varies, each element of the reduced model is assigned to align with the centriodal axis of the antenna mast. Consequently an element stiffness matrix of the reduced-order model differs from that of the standard Euler-Bernoulli beam. This also causes much error in the reduced model. In this regard, during the model tuning, the mass matrix is assumed correct and the element stiffness matrices are adjusted. For each element stiffness matrix, two coefficients α_E and α_G are introduced as the updating parameters to simplify the calculation, i.e., α_E represents the modulus variation coefficient that is associated with all items of the element stiffness matrix, and α_G is associated with bending and rotational DOFs only. As a result, the updated element stiffness matrix can be expressed as

$$\mathbf{K}_U^e = \begin{bmatrix} \alpha_E\mathbf{K}_{11}^e & \alpha_E\mathbf{K}_{12}^e & \alpha_E\alpha_G\mathbf{K}_{13}^e & \alpha_E\alpha_G\mathbf{K}_{14}^e & \alpha_E\alpha_G\mathbf{K}_{15}^e & \alpha_E\mathbf{K}_{16}^e & \alpha_E\mathbf{K}_{17}^e & \alpha_E\alpha_G\mathbf{K}_{18}^e & \alpha_E\alpha_G\mathbf{K}_{19}^e & \alpha_E\alpha_G\mathbf{K}_{110}^e \\ & \alpha_E\mathbf{K}_{22}^e & \alpha_E\alpha_G\mathbf{K}_{23}^e & \alpha_E\alpha_G\mathbf{K}_{24}^e & \alpha_E\alpha_G\mathbf{K}_{25}^e & \alpha_E\mathbf{K}_{26}^e & \alpha_E\mathbf{K}_{27}^e & \alpha_E\alpha_G\mathbf{K}_{28}^e & \alpha_E\alpha_G\mathbf{K}_{29}^e & \alpha_E\alpha_G\mathbf{K}_{210}^e \\ & & \alpha_E\alpha_G\mathbf{K}_{33}^e & \alpha_E\alpha_G\mathbf{K}_{34}^e & \alpha_E\alpha_G\mathbf{K}_{35}^e & \alpha_E\alpha_G\mathbf{K}_{36}^e & \alpha_E\alpha_G\mathbf{K}_{37}^e & \alpha_E\alpha_G\mathbf{K}_{38}^e & \alpha_E\alpha_G\mathbf{K}_{39}^e & \alpha_E\alpha_G\mathbf{K}_{310}^e \\ & & & \alpha_E\alpha_G\mathbf{K}_{44}^e & \alpha_E\alpha_G\mathbf{K}_{45}^e & \alpha_E\alpha_G\mathbf{K}_{46}^e & \alpha_E\alpha_G\mathbf{K}_{47}^e & \alpha_E\alpha_G\mathbf{K}_{48}^e & \alpha_E\alpha_G\mathbf{K}_{49}^e & \alpha_E\alpha_G\mathbf{K}_{410}^e \\ & & & & \alpha_E\alpha_G\mathbf{K}_{55}^e & \alpha_E\alpha_G\mathbf{K}_{56}^e & \alpha_E\alpha_G\mathbf{K}_{57}^e & \alpha_E\alpha_G\mathbf{K}_{58}^e & \alpha_E\alpha_G\mathbf{K}_{59}^e & \alpha_E\alpha_G\mathbf{K}_{510}^e \\ & & & & & \alpha_E\mathbf{K}_{66}^e & \alpha_E\mathbf{K}_{67}^e & \alpha_E\alpha_G\mathbf{K}_{68}^e & \alpha_E\alpha_G\mathbf{K}_{69}^e & \alpha_E\alpha_G\mathbf{K}_{610}^e \\ & & \text{Symmetric} & & & & \alpha_E\mathbf{K}_{77}^e & \alpha_E\alpha_G\mathbf{K}_{78}^e & \alpha_E\alpha_G\mathbf{K}_{79}^e & \alpha_E\alpha_G\mathbf{K}_{710}^e \\ & & & & & & & \alpha_E\alpha_G\mathbf{K}_{88}^e & \alpha_E\alpha_G\mathbf{K}_{89}^e & \alpha_E\alpha_G\mathbf{K}_{810}^e \\ & & & & & & & & \alpha_E\alpha_G\mathbf{K}_{99}^e & \alpha_E\alpha_G\mathbf{K}_{910}^e \\ & & & & & & & & & \alpha_E\alpha_G\mathbf{K}_{1010}^e \end{bmatrix} \tag{6}$$

where $\mathbf{K}_U^e$ and $\mathbf{K}^e$ are the updated and initial element stiffness matrices, respectively. *lsqnonlin* in MATALB was employed to minimise the objective function Eq. (5) under the constraints that $0.1 \le \alpha_E, \alpha_G \le 10$.

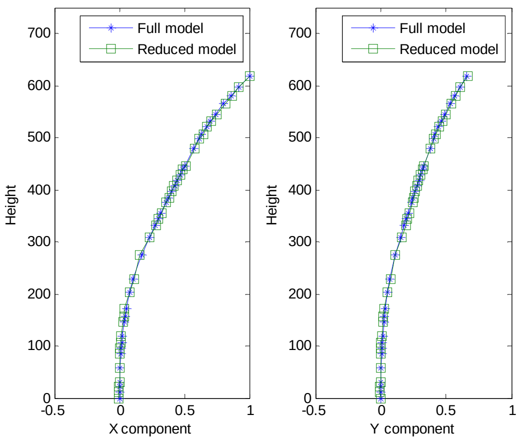
Model Tuning Results

The dynamic characteristics obtained from the full model and from the updated reduced model are listed in Table 2. Mode shapes of the first fifteen modes are compared in Fig. 4. One can see that the dynamic characteristics of the two models are in very good agreement.

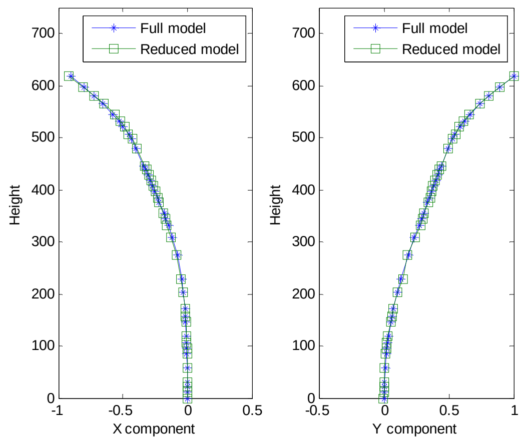
Table 2. Dynamic characteristics of the full model and reduced model

Mode	Frequency (Hz)			MAC (%)
	Full model	Reduced model	Difference (%)	
1	0.110	0.110	0.42%	99.98%
2	0.159	0.159	0.19%	99.97%
3	0.347	0.347	0.10%	99.53%
4	0.368	0.368	0.13%	99.52%
5	0.400	0.399	0.16%	99.55%
6	0.461	0.460	0.13%	99.86%
7	0.485	0.485	0.02%	99.39%
8	0.738	0.738	0.02%	99.29%

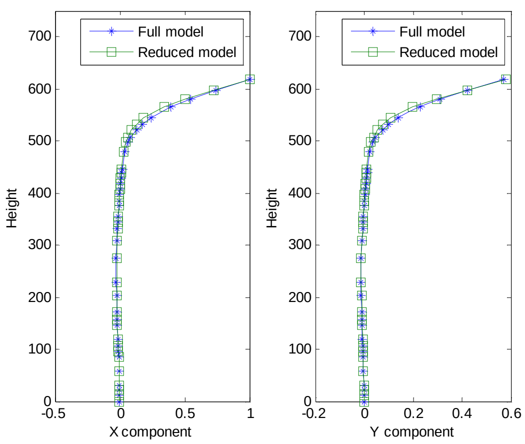
9	0.902	0.902	0.05%	99.36%
10	0.997	0.997	0.02%	99.43%
11	1.038	1.038	0.03%	98.99%
12	1.122	1.122	0.02%	99.41%
13	1.244	1.244	0.03%	98.31%
14	1.503	1.503	0.00%	96.76%
15	1.726	1.726	0.01%	97.50%



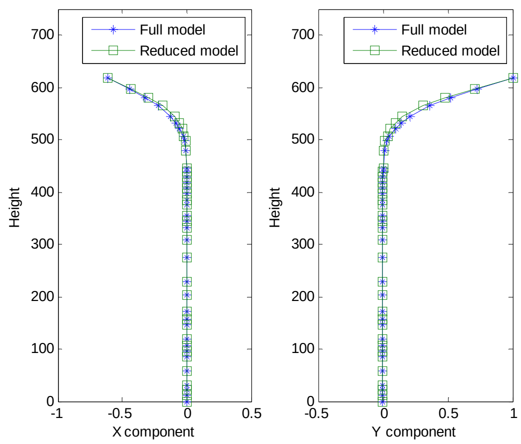
Mode 1: 1st short-axis bending



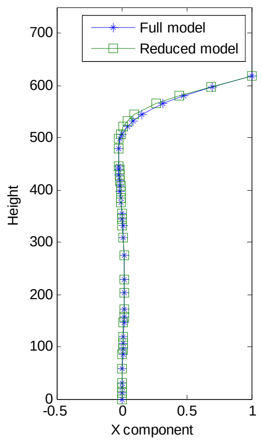
Mode 2: 1st long-axis bending



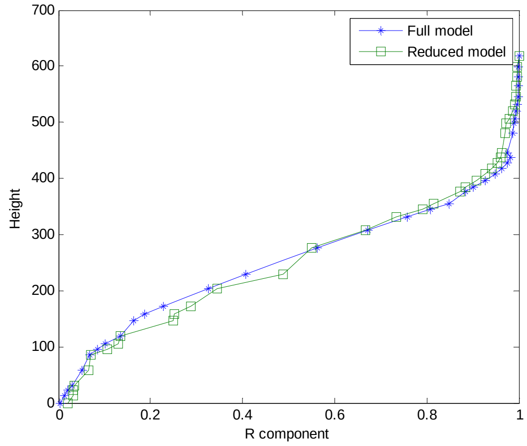
Mode 3: 1st short-axis bending of the mast



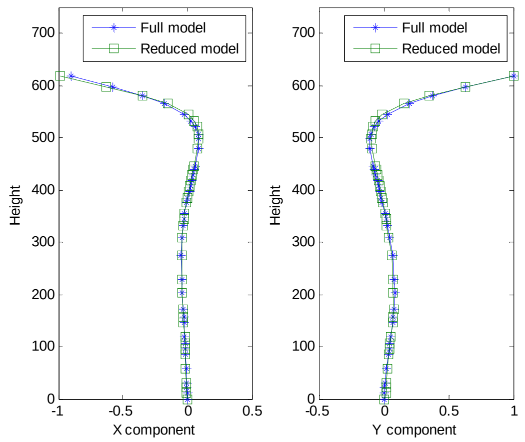
Mode 4: 1st long-axis bending of the mast



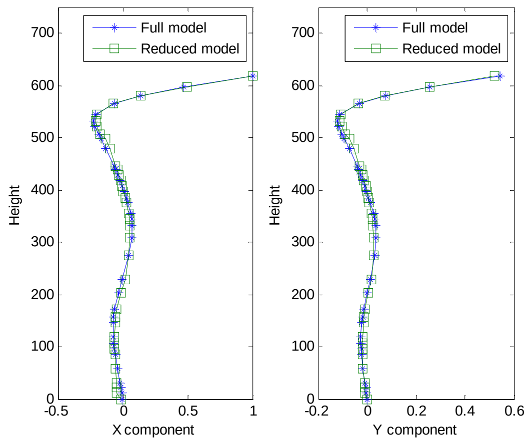
Mode 5: 2nd short-axis bending and 1st short-axis bending of the mast



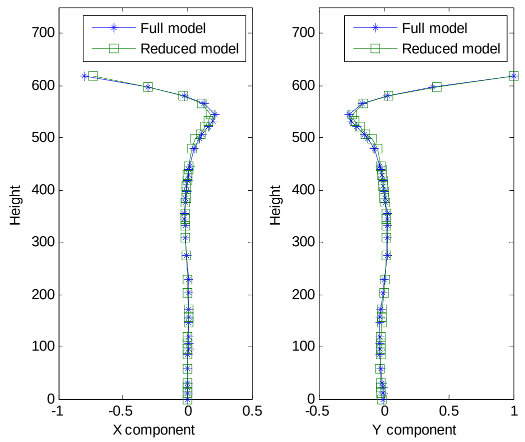
Mode 6: 1st torsion



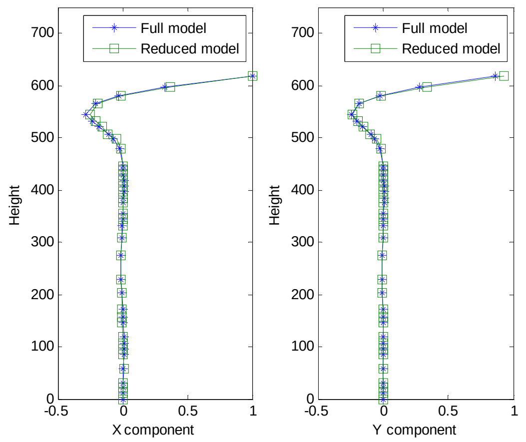
Mode 7: 2nd long-axis bending and 1st short-axis bending of the mast



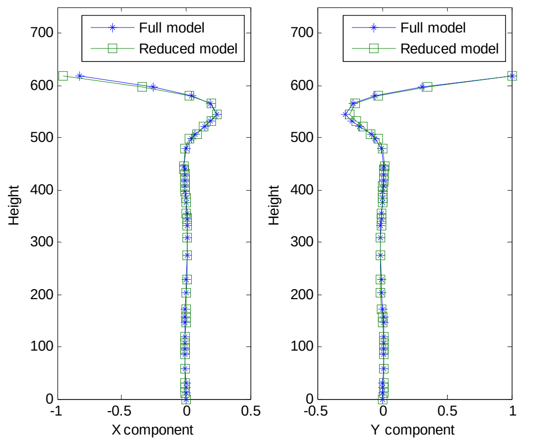
Mode 8: 3rd short-axis bending and 2nd short-axis bending of the mast



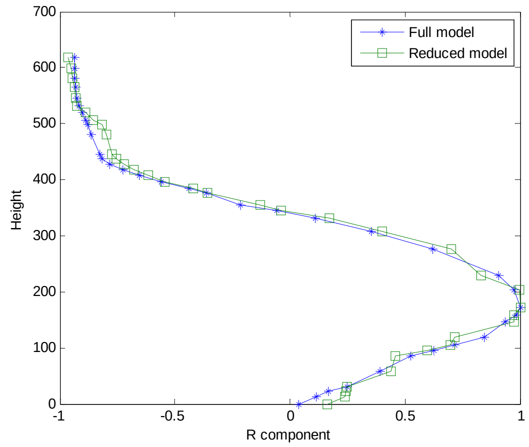
Mode 9: 3rd long-axis bending and 2nd long-axis bending of the mast



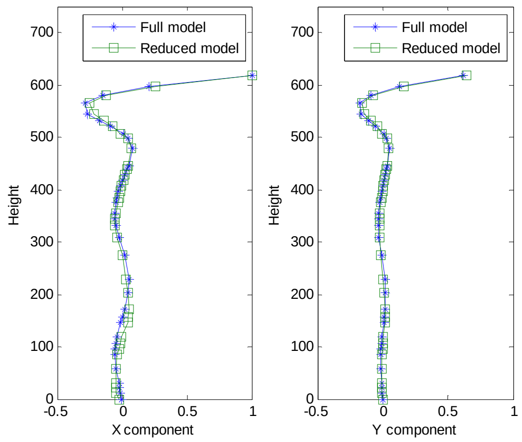
Mode 10: 2nd short-axis bending of the mast



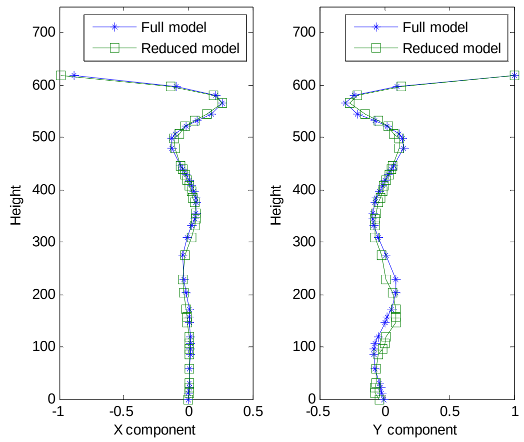
Mode 11: 2nd long-axis bending of the mast



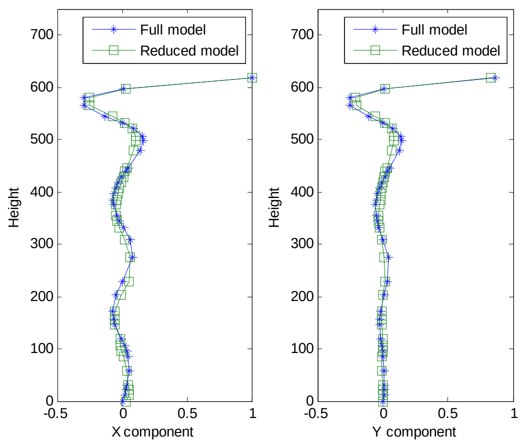
Mode 12: 2nd torsion



Mode 13: Coupled bending and torsion



Mode 14: Coupled bending and torsion



Mode 15: Coupled bending and torsion

Fig. 4 Comparison of the mode shapes between the full model and reduced model

Data Description

The element mass and stiffness matrices and the global mass and stiffness matrices of the updated reduced model are stored in MATLAB format (.mat, version 6.5), and can be downloaded at http://www.cse.polyu.edu.hk/benchmark/Phase_I_data/system_matrices.mat. The MAT file includes 76 variables: kk_??, kk_global, mm_??, and mm_global, where 'kk' denotes the stiffness matrices, 'mm' denotes the mass matrices, '??' are from 01 to 37 and represent the corresponding element numbering, and 'global' represents the global matrices.

Apart from the first mass and stiffness element matrices with a size of 5×5 each, all other mass and stiffness element matrices have a size of 10×10. In each element matrix, indices 1 to 5 refer to the lateral translation DOFs in *x* and *y* directions, and rotations along *x*, *y* and *z* axes at the lower node, respectively. Similarly indices 6 to 10 refer to the DOFs at the upper node. The global matrices have a size of 185×185. The *i*th node (i = 2, …, 38) contributes to the DOFs between (i-2)×5+1 and (i-2)×5+5 of the global matrices.

It is noted that principal SI units are used for all matrices elements, i.e., *kg* for mass, *N* for force, *m* for translation displacement, and *rad* for rotation.